Experiment No. 5

The Synchronous Machine

Synchronous ac machines find application as motors in constant speed applications and, when interfaced to the power source with a variable-frequency converter system, in variable-speed applications. High-performance variable-speed motor drives are often constructed using a permanent-magnet synchronous motor. Another common application is the alternator that generates the power for the automobile electrical system. In addition, practically all electrical energy produced commercially is generated in rotating synchronous machines that are connected to the utility or other power systems. Most power systems are supplied by a number of such machines operating in parallel, and the systems themselves are interconnected to form power grids of tremendous energy capacity. In comparing the capability of any single generator to the overall capacity of the system to which it is connected, it can be seen that the individual machine is relatively insignificant, even though its own power rating may be in the neighborhood of one million kilowatts. In other words, it can be thought of as being connected to a voltage bus of infinite capacity, so that regardless of how much energy it delivers to (or receives from) the system, the voltage and frequency remain constant.

The objectives of this experiment are (1) to develop a basic model for the synchronous machine, (2) to apply the model to understand the characteristics of the machine when connected to an infinite bus, and (3) to synchronize an ac generator to a large power system.

1. An equivalent circuit model for the synchronous machine

A basic synchronous machine is sketched in Fig. 1. The stator contains a three-phase armature winding. When a source of three-phase ac is connected to this winding, a magnetic field of constant amplitude is produced within the machine; in a two-pole machine, this field rotates at frequency equal to the frequency of the applied ac. The rotor contains a field winding that is excited by dc; this winding behaves as an electromagnet, producing a field of strength proportional to the applied field current that is aligned with the axis of the field winding. Alternatively, the field winding may be replaced by a permanent magnet. Torque is produced by the two magnetic fields attempting to align, according to the formula

$$T = F_s F_r \sin\left(\delta\right) \tag{1}$$

where F_s and F_r are the magnitudes of the stator and rotor fields, respectively. The angle δ is the angle between the stator and rotor fields, commonly called the *torque angle*.



The

equivalent circuit of Fig. 1 the synchronous machine closely resembles the dc machine model used

in Experiment 1. The key difference is the absence of commutator brushes, such that the armature voltage and current are ac. Also, the field winding is normally placed on the rotor with the armature winding on the stator.

The magnitude of the magnetic field produced by the rotor field winding, F_r , is proportional to the applied field current I_f :

$$F_r = \frac{N_f I_f}{\Re} \tag{2}$$

where N_f is the number of turns in the field winding, and \Re is the total reluctance in the magnetic path of the rotor flux.

When the shaft turns, the rotor field induces a voltage in

A synchronous machine. Three-phase stator (armature) windings are not explicitly shown. Torque is produced when the magnetic field of the dc rotor winding (aligned with the rotor axis) attempts to align with the rotating magnetic field of the threephase ac stator winding.



Fig. 2 Equivalent circuit model of the synchronous machine.

the armature (stator) winding. This induced voltage is similar to the back EMF of the dc machine, except that it is an ac voltage having a magnitude E and phase δ with respect to the applied stator voltage. The magnitude E is proportional to the rotor field strength F_r and the shaft speed ω :

$$E = KF_r \omega \tag{3}$$

where K is a constant of proportionality that depends on the machine and winding geometry. ω is the angular shaft frequency. The stator winding model therefore includes a voltage source $E \angle \delta$ (phasor notation) as in Fig. 2. In addition, a Thevenin-equivalent model of this winding contains a series impedance consisting of a series inductance,

known as the synchronous inductance L_s , and a series resistance R_s that models the resistance of the wire.

2. Solution of the equivalent circuit, connected to an infinite bus

The circuit of Fig. 2 models one (line-to-neutral) phase of the three-phase machine. The applied voltage at the terminals of the stator winding is taken to be $V \angle 0$. In a balanced three-phase system, all three phases have symmetrical waveforms. The mechanical output power $T\omega$ is then equal to three times the average per-phase electrical power flowing into the voltage source E:

$$T\omega = P = 3\text{Re}(\boldsymbol{EI_s}^*)$$
(4)

In Eq. (4), E is the phasor $E \angle \delta$, I_s is the phasor representing the stator current, I_s^* is the complex conjugate of the stator current,



Fig. 3 A typical generator application. Field winding is excited by dc current source I_f . Armature is connected to three-phase ac infinite bus, modeled by voltage source (one phase shown). Shaft is connected to a source of mechanical power, called the "prime mover."

and $\operatorname{Re}(EI_s^*)$ is the average power flowing into the voltage source *E*.

Figure 3 consists of the equivalent circuit of Fig. 2, connected to an infinite bus having voltage $V = V \angle 0$. For simplicity, the stator winding resistance R_s has been neglected. When the synchronous machine is connected to infinite bus V, the rotor must turn at angular frequency ω equal to the angular frequency of the infinite bus. However, the rotor can be shifted in phase (by angle δ). Let us determine the stator winding current I_s and the average power $3\text{Re}(EI_s^*)$. Solution of the equivalent circuit of Fig. 3 to find I_s leads to

$$I_s = \frac{E - V}{j\omega L_s} \tag{5}$$

Substitution of $V = V \angle 0$ and $E = E \angle \delta$ leads to

$$I_{s} = \frac{E\cos\left(\delta\right) + jE\sin\left(\delta\right) - V}{j\omega L_{s}}$$
(6)

The average power is then

$$P = 3\operatorname{Re}\left\{ EI_{s}^{*}\right\}$$

$$= 3\operatorname{Re}\left\{ \left(E\cos\left(\delta\right) + jE\sin\left(\delta\right)\right) \left(\frac{E\cos\left(\delta\right) - jE\sin\left(\delta\right) - V}{-j\omega L_{s}}\right)\right\}$$

$$= 3\frac{VE\sin\left(\delta\right)}{\omega L_{s}}$$
(7)

The generator loads the mechanical shaft with torque

(8)

$$T = \frac{P}{\omega} = 3 \frac{VE\sin(\delta)}{\omega^2 L_s}$$

This equation is plotted in Fig. 4. For given values of V and E, there is a maximum torque T_{max} that the machine can produce, which occurs at $\delta = 90^{\circ}$. As the power and torque of the generator are increased, the torque angle increases and the



Fig. 4 Torque vs. torque angle characteristic of the cylindrical rotor synchronous machine, from Eq. (8).

rotor leads the stator rotating field. It should be noted that the above equations and Fig. 4 are valid for a *cylindrical rotor* machine, in which the length of the air gap is uniform. In a *salient pole* machine (such as Fig. 1), the rotor is not cylindrical and the air gap length depends on the angle with respect to the rotor axis. The expression for torque and power is more complex in a salient pole machine, and the maximum torque typically occurs at a smaller value of δ .

If a transient causes the instantaneous torque angle to exceed 90°, then the machine will "slip a pole" and the torque angle will continue to increase. If the prime mover torque is increased beyond T_{max} , then the shaft will accelerate, synchronism is lost, and the speed may increase beyond a safe value. In either event, large transient torques can occur that may damage the shaft and machine.

3. Discussion

When the synchronous machine is connected to an infinite bus, the shaft speed is determined by the frequency of the infinite bus and is independent of other quantities such as field currents, load torque, etc. To the extent that the synchronous machine is ideal, its mechanical input power must be equal to its electrical output power. To change the electrical output power, the energy source supplying mechanical input power to the machine must be changed. For example, if the energy source is a gasoline engine, then to increase the electrical output power of the generator the engine throttle must be opened, burning more gasoline. In our laboratory experiment, the mechanical energy source will be a dc motor. To increase the power, the dc motor field current will be reduced; this causes the dc motor to

generate more mechanical torque and transmit more mechanical power down the shaft. Note that changing the field current of the ac generator has no effect on the power flow.

Then what is the effect of changing the field current? Figure 5 illustrates what happens



Fig. 5 Effect of changing the field current I_f . Prime mover power is kept constant and equal to P_1 .

when I_f is reduced while the average power is maintained constant at value $P = P_1$. Initially, the machine operates with large field current; the torque angle δ_a is found by solution of Eq. (7). When the field current is reduced, the magnitude of the induced voltage E is reduced proportionally, according to Eqs. (2) and (3). To maintain power P_1 , the torque angle must increase to δ_b . Note that decreasing I_f also causes P_{max} and T_{max} to decrease, decreasing the stability margin.

Figure 6 illustrates the effect of reducing I_f on the armature current I_s and power factor. The phasor diagram of V, E, and I_s is constructed using Eq. (5). Figure 6(a) represents the original case with large I_f and with $\delta = \delta_a$. The voltage E induced in the stator winding by the rotor field has relatively large magnitude, and leads the infinite bus voltage V by angle δ_a . The voltage across the synchronous inductance L_s is equal to $(E - V) = j\omega L_s I_s$. We can find I_s by dividing this voltage by $j\omega L_s$; I_s lags (E - V) by 90°. For the example sketched



Fig. 6 Phasor diagram of armature voltages and current, Eq. (5): (a) overexcited case, generator supplies real and reactive power, I_{ss} lags V. (b) underexcited case, same real power as case (a) but I_f has been reduced. Generator supplies real power but consumes reactive power, I_s leads V.

in Fig. 6(a), I_s lags V and hence the generator supplies both real power and reactive power to the infinite bus. The angle θ is the phase of I_s with respect to V, and hence the power factor is $\cos \theta$. When I_s lags V, the generator is said to be *overexcited*.

The *underexcited* case is illustrated in Fig. 6(b). The field current I_f has been reduced, with constant power $P = P_1$, and hence the torque angle is now $\delta = \delta_b$. The magnitude *E* is also reduced, as described above. This changes the voltage (E - V) as shown, and hence I_s now leads *V*. The generator now consumes reactive power.

The effect of field current I_f on the magnitude of the armature current $|| I_s ||$ is summarized by Fig. 7. For a given average power P, the minimum armature current $|| I_s ||$ is observed at the value of I_f that leads to unity power factor. Increasing I_f above the unity power factor setting causes generation of reactive power with the same real power; hence $|| I_s ||$ is increased. Decreasing I_f below the unity power factor setting causes consumption of reactive power with the same real power; hence $|| I_s ||$ is again increased.



Fig. 7 Generator "V-curves": armature current magnitude vs. field current for given values of average power *P*.

A number of other systems can be modeled using the armature equivalent circuit of Fig. 3. For example, the voltage source inverter circuit is the most commonly-used power electronics approach to interfacing a dc power source to the ac line; this circuit can be modeled as an effective voltage source behind an inductor. Another example is the transmission of power through a transmission line, from one voltage bus to another. In each case, Eqs. (5) through (8) apply, with a result similar to Figs. 5 and 6.

4. Synchronizing to an infinite bus

Let's consider next how to connect a generator to an infinite bus. When preparing to operate a generator (either ac or dc) in parallel with other generators or a power system, it is always necessary to make certain that no potential difference exists across the terminals of the paralleling switch when it is closed. It must be remembered that both the machine and the ac system represent very low impedance circuits, and even a small potential difference can result in very large circulating currents that may cause unwanted circuit breaker openings and/or damaging shaft torques.

When paralleling three-phase ac generators, three voltmeters could be used: on across the switch in each phase. When all three voltmeters read zero, the switches are closed to connect the generators. Instead of voltmeters, light bulbs are often used; when all three light bulbs are dark, the voltages across the switches are close to zero.

The procedure to synchronize a generator to a power system is therefore as follows:

- (a) Drive the generator at the speed that causes its stator winding frequency to be approximately equal to the power system frequency
- (b) Make sure that the phase sequences of the generator and power system are the same
- (c) Set the generator voltage to be equal to the power system voltage, by adjusting the generator field current
- (d) Wait until all three voltages across the switches are simultaneously zero (or until all three light bulbs are dark), then close the switches.

When the switches are closed, a transient will pull the rotor into exact synchronism with the power system. If the paralleling procedure is performed correctly, then this transient will be small in magnitude. In large installations, synchronization is performed using a *synchroscope* that displays the phase difference between the generator and power systems. New designs often contain controllers that synchronize automatically.

PROBLEMS

1. A hydroelectric generator

A certain three-phase synchronous generator, designed to operate at a hydroelectric generating station (such as Hoover Dam), has the following ratings:

shaft speed	120rpm
electrical frequency	60Hz
power	60MW, 90MVA
voltage	26kV

You are the engineer responsible for going to the dam and operating the generator for the first time.

- **Synchronizing to the western power grid:** The water gates are opened a little bit, and the machine is brought up to speed. The dc field current is adjusted to 50A, which produces a line-line voltage of 26kV, matching the power system voltage magnitude. The phase sequence and phase are matched, and then the big switch is thrown to connect to the power grid. The generator then produces zero watts and zero VARs.
- (a) What is the torque angle under these conditions?

- **Next**, the central dispatch office tells you to increase the generated power to 30MW at unity power factor. So you open up the water gates to obtain a 30MW output power, simultaneously increasing the field current to 60A, such that unity power factor is obtained.
- (b) Determine
 - (i) the per-phase synchronous inductance L_s , and
 - (ii) under these conditions, the torque angle δ .
- **Next,** the central dispatch office tells you to supply 10MVARs (lagging), while maintaining the same real power of 30MW.
- (c) (i) How should you change the field current?
 - (ii) What is the new torque angle?
- **2.** A three-phase 60 Hz synchronous generator is operating at unity power factor, with the following conditions:

line-line voltage	13kV
stator current	1000A
field current	80A
torque angle	20°

(a) Determine:

(i) the magnitude of the induced voltage E, and

(ii) the value of the synchronous inductance L_s .

(b) It is now desired to supply 8MVARs (lagging) while maintaining the same real power as above. Sketch the phasor diagram showing E, V, and I for this case, and determine numerical values for (i) the new field current, and (ii) the new torque angle δ .

Experiment 5 Pre-lab assignment

ECEN 4517 / 5017

Synchronous machine

- **1.** Read all sections of the text.
- 2. Do problem 1
- **3.** Read the laboratory procedure

This assignment is due from each student at the beginning of the lab session.